

ON THE STRENGTH OF A BODY AT TEMPERATURES CLOSE TO ITS MELTING POINT

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On the example of frozen soils and ice existing at temperatures close to the melting point, the existence of two mechanisms (types) of destruction related to the phase transitions of water is shown. The first mechanism that operates near the melting point is plastic destruction, it gradually converts an ice body into a liquid-like state. The second mechanism operating far from the melting point is brittle destruction, disintegration into parts, with a slight preliminary deformation. For both types of destruction equations of long-term strength and of the behavior of deformation in time have been obtained.

Keywords: frozen soil, ice, plastic destruction, brittle destruction, deformation, liquid-like state, long-term strength, longevity.

Introduction. According to the kinetic concept of strength, the equation of longevity of a solid body has the form [1]

$$\tau_{l,t} = \tau_a \exp [(U - \gamma P)/RT]. \quad (1)$$

Equation (1) was obtained empirically by generalizing numerous breaking tests with various materials, including composite and heterogeneous ones. Nevertheless, judging by the experimental data of [2–4], it is not appropriate for the description of the longevity of ice and frozen soils. For example, in breaking tests with ice samples at $T = 295$ K and P equal to 1.0, 0.90, 0.85, and 0.80 MPa, the value of $\tau_{l,t}$ was equal to 10, 25, 50, and 150 min, respectively [3]. At the same time, calculations by Eq. (1) even at the same T and $P = 0$ yield only $\tau_{l,t} = 2.6 \cdot 10^{-3}$ sec. Moreover, according to Eq. (1), the minimum longevity $\tau_{l,t} = \tau_a \approx 10$ sec is attained at $T = \infty$. However, the solid body temperature cannot be higher than the melting temperature (T_{mel}). Consequently, Eq. (1) is applicable only to materials with a very high T_{mel} , for example, to metals whose melting temperature at an atmospheric pressure ($T_{mel} = T_0$) attains 1500–2000 K and above. Ordinary ice (ice I) melts at temperatures 6–8 times smaller ($T_{mel} = 250$ –273 K). Equation (1) is based on the concept of gradual accumulation of the disturbances (deformations) of the structure of a solid body, culminating in its destruction. Deformations can be external, observable, expressed as a change in the shape and volume, and internal, hidden ones. The latter are mainly connected with the process of cracking that almost does not influence the volume and shape of a body, but on opening of cracks, causing its disintegration into parts — the destruction proper. According to this, at the atomic level the disturbances are understood [1] to be ruptures of individual atomic bonds in the process of their thermal vibrations with the escape of atoms beyond the crystal lattice as a result of large thermal fluctuations, which are random energy outbursts up to a value equal to the sublimation energy Q_{sub} . This quantity is taken to be the activation energy of destruction $U \approx Q_{sub}$.

The higher the temperature T and pressure P , the more frequent the thermal fluctuations and the more rapidly the destruction occurs. According to Eq. (1), destruction may occur also in the absence of loading, when $P = 0$, i.e., there exists (and can be determined) a maximum possible longevity of a body τ_{max} :

$$\tau_{max} = \tau_a \exp (Q_{sub}/(RT)) = \tau_a \exp (Q_{sub}T_0/T)/(RT_0).$$

For ordinary ice $Q_{sub}/(RT_0) = 24$. For many metals this ratio, meaning the number of thermal fluctuations, on the attainment of which a body is destroyed even at $P = 0$, is also close to 24: 25 for zinc, 30 for copper, 31 for aluminum, 27 for silver, etc. As to the coefficient T_0/T , then for ice I existing in the range $T = 251$ –273 it is close

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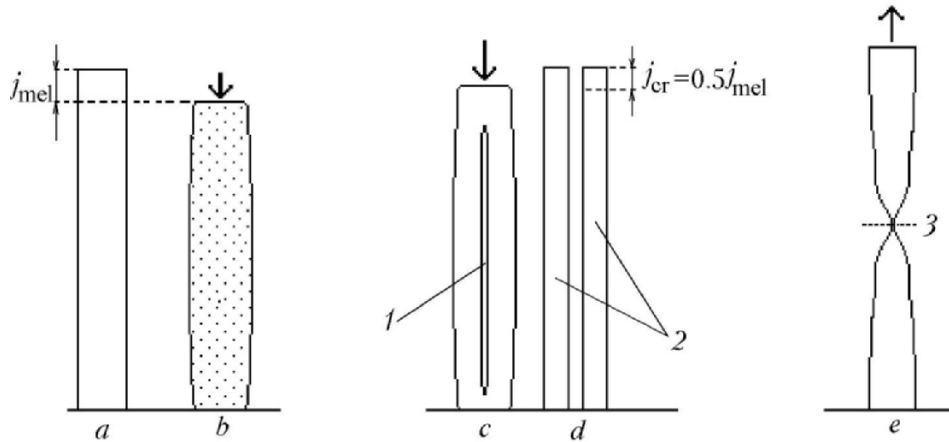


Fig. 1. An approximate scheme of action of various mechanisms of destruction of a frozen soil: a) prismatic fragment of soil before loading; b) its "liquefaction" after loading; c) formation of cracks; d) elastic disintegration; e) destruction on extension; 1) crack; 2) disintegration of a fragment; 3) transverse rupture plane.

to unity. It is several times higher for metals (for example, for iron at the same temperatures it is close to 7). Hence, the unlikely small longevity of ice calculated by Eq. (1): $(2.6-2.9) \cdot 10^{-3}$ sec, which is at variance with the actual data.

We may also relate thermal fluctuations with energy equal to the melting heat Q_{mel} to rupturing ones. At such small thermal fluctuations, despite the fact that the atoms remain within the crystal lattice, they deviate from their previous position to such an extent that they already cannot return into it. Their bonds become weaker, they lose the elasticity, and residual deformations appear and begin to prevail. The maximum value of the returning (elastic) deviations of atoms in all of the substances usually does not exceed 10%, i.e., at the macrolevel it corresponds to the relative deformation of melting $j_{mel} = |V_{sol} - V_{liq}| / V_{sol}$. The accumulation of residual deformations gradually transforms the solid body into a qualitatively different state — a liquid-like, flowing state, with the volume $V \approx V_{liq}$, which can also be considered as its conventional destruction, not a brittle, rapid one, but rather an extended in time, plastic one. The criterion of such a state is the attainment of j_{mel} . Sublimation corresponds to the ultimate rupture of the atomic bond, to the formation of a "void" in its place, when the relative deformation $j_{mel} \approx 1$. At the macrolevel such a deformation is local, adapted to portions with weakened bonds — stress concentrators (microcracks, vacancies, etc.) develop in the form of a crack in the direction of compression. The longevity of a body is equal to the total time of rupture of all atomic bonds that connect the walls of the crack.

Two Mechanisms of Destruction, General Equations of Strength and Longevity. A frozen soil represents a set of organomineral particles with inclusions of an unfrozen water (solutions) and gases connected by porous ice. The strength of cohesion of its particles is ensured, just as for ice, mainly by ice bonds [2]. The remaining ones can be neglected in the first approximation and the liquefying of the ice bonds with subsequent spreading of the soil can be evaluated as plastic destruction and j_{mel} as its limiting deformation. Thus, we may speak about two basic mechanisms (types) of destruction of a frozen soil: a slow, plastic one that develops near the melting point, and a brittle (sublimation) one far from it. Figure 1 shows a simplified (basic) diagram of the action of these mechanisms.

Equation (1) describes brittle destruction, i.e., at the macrolevel, that is, the process of formation of a crack up to the moment of its opening. For engineering calculations it is advisable to simplify Eq. (1) by discarding the microparameters. At $\tau_{1,t} = \tau_a$ the instantaneous (maximum) strength $P_{m.sub} \approx Q_{sub} / \gamma$. An analysis of the reference data [5] has shown that for all of the substances the ratio RT_0 / Q_{sub} is close to a constant value that will be presented in the form $j_{cr} \approx \eta j_{mel}$, and we will endow it with the meaning of the volumetric relative deformation of a body at the moment of crack opening ($\eta = RT_0 / (Q_{sub} j_{mel}) = RT_0 / Q_{mel}$ is the coefficient of averaging). Then Eq. (1) is transformed into

$$\tau_{1,t} = \tau_a \exp [(P_{m.sub} - P) / (P_{m.sub} j_{cr})] . \quad (2)$$

The value of the coefficient η lies between zero and unity. Calculations show that in the majority of solid bodies this value does not differ greatly from 0.5: for ice it lies within 0.5–0.53, for zinc it is equal to 0.57, for bismuth and lead to 0.64, and for tin and aluminum to 0.46, i.e., the ratio between RT_0 and Q_{mel} is approximately the same as for the energy and work of elastic forces in Hooke's law. Therefore it is admissible to interpret the mean energy of the vibrations of atoms RT_0 as the elasticity (reversibility) energy, the melting heat $Q_{\text{mel}} \approx 2RT_0$ as the work of elastic forces, and the sublimation heat Q_{sub} as the total energy of bonds equal to the work of their destruction (rupture). An equation of the type of Eq. (2) transfers the calculation of the longevity from the microlevel to the macrolevel, from the hidden (internal) process of the formation of cracks to a simultaneously occurring, observed process of general deformation of a body. *Far from the melting point*, the crack opens earlier than the bulk deformation j attains its limiting value j_{mel} , approximately at $j = j_{\text{cr}} \approx \eta j_{\text{mel}} \approx 0.5j_{\text{mel}}$; the body disintegrates into parts with the restoration, in them, of the shape and initial specific volume V_{sol} . This is brittle destruction. *Near the melting point*, the body under pressure is liquefied, its internal voids are filled in, and cracks are not formed. At $j = j_{\text{mel}}$ a body converts into a liquid-like state. This is plastic destruction.

For a full description of the behavior of a loaded solid body, one has also to know the dependence of deformation on time. Having taken from (2) the derivative j_{cr} with respect to $\tau_{1,t}$, we find an equation for the rate of deformation, having integrated j in which from 0 to j_{cr} and τ from τ_a to $\tau_{1,t}$, we obtain, after simple transformations, the sought expression of the time dependence of deformation (average over the entire volume) for a short-brittle body:

$$j/j_{\text{cr}} = \ln(\tau/\tau_a) / (\ln(\tau_{1,t}/\tau_a)). \quad (3)$$

The ratio of the finite difference $P_{\text{m.sub}} - P = \Delta P$ to $P_{\text{m.sub}}$ in (2) corresponds to the condition of a jumpwise change in the internal pressure from the value of the external P to a maximum of $P_{\text{m.sub}}$. However, in the case of plastic destruction, the internal pressure successively passes the entire spectrum of values — from the external P to a maximum $P_{\text{m.mel}}$, whose value is determined in this case from the classical Clapeyron–Clausius equation: $P_{\text{m.mel}} = Q_{\text{mel}}/T_0(V_{\text{sol}} - V_{\text{liq}})$. Correspondingly, the longevity changes from $\tau_{1,t}$ to τ_a . Therefore on plastic destruction, in Eq. (2) the ratio $\Delta P/P_{\text{m.sub}}$ should be replaced by the integral of dP/P from P to $P_{\text{m.mel}}$, and $\tau_a/\tau_{1,t}$ by the integral of $d\tau/\tau$ from $\tau_{1,t}$ to τ_a . Moreover, the observed limiting deformation is equal in this case to j_{mel} rather than j_{cr} (η is equal to unity and not to 0.5). After these transformations of Eq. (2), we obtain an expression for the long-term strength of a plastic body:

$$P = P_{\text{m.mel}} (\tau_a/\tau_{1,t})^{j_{\text{mel}}}. \quad (4)$$

Having taken in (4) the derivative of j_{mel} with respect to $\tau_{1,t}$, we find the equation for the rate of deformation, and having integrated j from 0 to j_{mel} and τ from τ_a to $\tau_{1,t}$, we obtain, after simple transformations, an expression for the time dependence of deformation for a plastically rupturing body:

$$j/j_{\text{me}} = \ln(\tau/\tau_a) / (\ln(\tau_{1,t}/\tau_a)). \quad (5)$$

Equations (3) and (5) differ only in the values of the limiting volume deformation j_{cr} and j_{mel} , the latter being approximately twice as large. The strength as well as deformation are present in both quantities.

Equation of Phase Equilibrium and Long-Term Strength. Equations for a long-term strength and time dependence of the deformation of a frozen ground and ice can be derived also directly from the generalized Clapeyron–Clausius equation [6], which will be written in the form [7]

$$t_{\text{mel}} = P(V_{\text{sol}} - V)T_0/Q_{\text{mel}} = bP. \quad (6)$$

The parameter $b = (V_{\text{sol}} - V_{\text{liq}})T_0/Q_{\text{mel}} = 0.917 - 0.83D$ °C/MPa in (6) has the meaning of the coefficient of commensurability of the pressure and melting temperature. Its value depends on the degree of compression (closeness) of an unfrozen water by ice crystals $D = P_{\text{por}}/P$. At $D = 1$ (when $P_{\text{por}} = P$ and $V = V_{\text{liq}}$, i.e., in the case of the classical limiting variant of the phase equilibrium equation), the system is completely closed, $b \approx 0.08$ °C/MPa; at $D = 0$ ($P_{\text{por}} = 0$, $V = 0$) the system is completely open, $b \approx 0.92$ °C/MPa. The processes of freezing-through and melt-

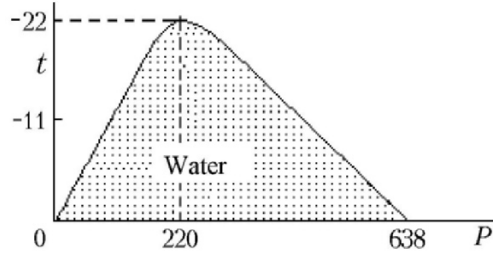


Fig. 2. Region of existence of ordinary water at $t < 0^\circ\text{C}$. t , $^\circ\text{C}$; P , MPa.

ing proceed predominantly at b close to $0.08^\circ\text{C}/\text{MPa}$. Higher b are noted in frozen soils not saturated with water, as well as in the case of their free moisture exchange with the surrounding medium (open systems).

Unlike the majority of other substances, the specific volume of water increases during freezing, whereas the freezing temperature decreases with increase in pressure. By not taking into account the signs of the change of these parameters, as follows from Eq. (1) the character of their interaction is the same in all of the substances: the absolute value of t_{mel} is the higher the greater the deviation of the pressure or volume from their initial values.

At approximately $t = -22^\circ\text{C}$ (251 K) and $P \approx 214\text{--}220$ MPa [8] the sign of the dependence of t_{mel} on P is reversed: the value of t_{mel} begins to increase, attaining 0°C at $P = 638$ MPa. At $t < -22^\circ\text{C}$ ordinary water, and at $P > 220$ MPa ordinary ice (ice I [8]), do not exist (Fig. 2). By analogy with oversaturated solutions (alloys), the temperature and pressure in this limit will be called eutectic. The specific heat of melting at the eutectic point is $Q_{\text{mel}} = 235$ kJ/kg (4.2 kJ/mole), and the volume one is $L_{\text{mel}} = Q_{\text{mel}}/V_{\text{sol}} = 216,000$ kJ/m³ = 216 MPa $\approx P_{\text{m.mel}}$; the volume heat of crystallization is $L_{\text{crys}} = Q_{\text{mel}}/V_{\text{liq}} = 235,000$ kJ/m³ = 235 MPa $\approx P_{\text{m.crys}}$; $L_{\text{mel}}/L_{\text{crys}} = 0.917$. Having multiplied the Planck constant, the average frequency of vibrations of atoms, and the Avogadro number, we find the molar (related to a mole) value of the energy quantum: 4 kJ/mole, which is almost equal to $Q_{\text{mel}} = 4.2$ kJ/mole (235 kJ/kg) at $t \approx -22^\circ\text{C}$ ($T_{\text{eut}} \approx 251$ K). At this very temperature $Q_{\text{sub}} = 2830$ kJ/kg [8], which amounts to $2830/235 = 12$ quanta. An analysis of the actual (reference) data has also shown that the spatial and thermal parameters that characterize the limits of existence of water and ice and of their deformations in phase changes are approximately similar:

$$V_{\text{liq}}/V_{\text{sol}} = T_{\text{eut}}/T_0 = Q_{\text{ev}}/Q_{\text{sub}} = P_{\text{m.mel}}/P_{\text{m.crys}} \approx 0.917, \quad (7)$$

$$1 - (V_{\text{liq}}/V_{\text{sol}}) = 1 - (T_{\text{eut}}/T_0) = Q_{\text{mel}}/Q_{\text{sub}} = 1 - (P_{\text{m.mel}}/P_{\text{m.crys}}) \approx j_{\text{mel}} \approx 0.083, \quad (8)$$

$$(V_{\text{sol}}/V_{\text{liq}}) - 1 = (T_0/T_{\text{eut}}) - 1 = (Q_{\text{sub}}/Q_{\text{ev}}) - 1 = (P_{\text{m.crys}}/P_{\text{m.mel}}) - 1 \approx j_{\text{crys}} \approx 0.091. \quad (9)$$

From Eqs. (6)–(9) the linearity of the distribution of t , P , and j between their extreme values and an approximate analogy of the set of these equation to the gas laws follow. We also note in (6) the coincidence of the upper and lower limits of the change in the coefficient $b = 0.917\text{--}0.83D$ related to the unity of their measurement, i.e., $(0.917^\circ\text{C}/\text{MPa}) / (1^\circ\text{C}/\text{MPa}) = 0.917$ and $(0.087^\circ\text{C}/\text{MPa}) / (1^\circ\text{C}/\text{MPa}) = 0.087$, with the similarity coefficients in Eqs. (7) and (8). Moreover, the ratios of the heats of sublimation and melting at $t = -22^\circ\text{C}$ to RT_0 are equal to $2830/(0.46 \cdot 251) = 24$ and $235/(0.46 \cdot 251) = 2$, respectively.

Equations (8) and (9) describe the relative deformations of volume and energy parameters in melting of ice and crystallization of water. All the expressions entering into Eq. (7) are equal to the ratio of the densities of ice and water (0.917/1). The maximum possible compression deformation of any substance is equal to unity. Consequently, the solid phase of water (initial in cooling) consists of $1/0.083 = 12$, whereas the liquid phase (initial in freezing) consists of $1/0.091 = 11$ parts (quanta) equal to $Q_{\text{mel}} = 235$ kJ/kg each. In freezing, one other quantum (the 12th one) is spent on hidden formation and appearance of crystallization nuclei.

We will write Eq. (6) separately for $D = 1$ and $D = 0$ in the form

$$P(V_{\text{sol}} - V_{\text{liq}}) = Q_{\text{mel}}t/T_0, \quad (10)$$

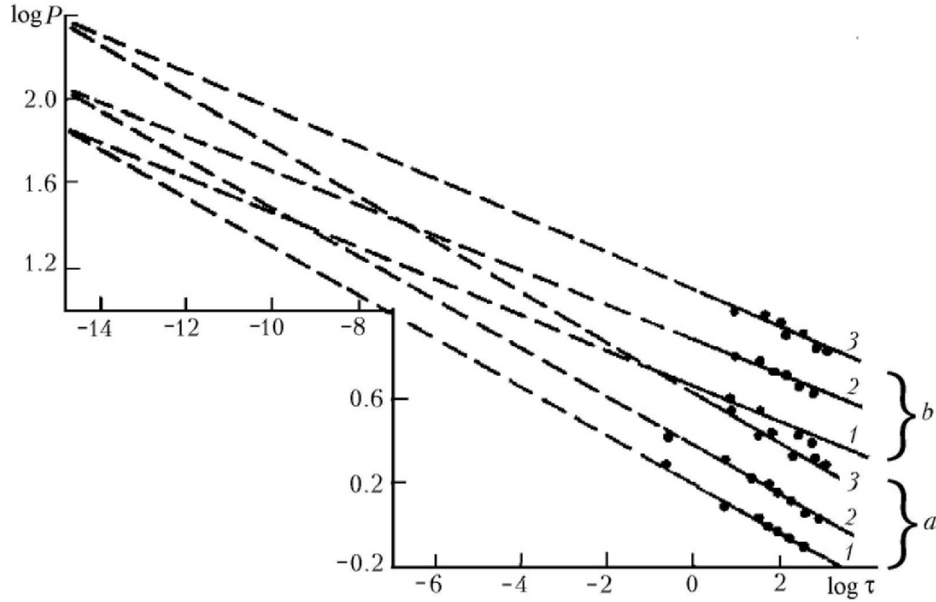


Fig. 3. Logarithmic dependence of long-term strength on time (a) compression of sand loam; b) cohesion of clay) at temperatures: 1, $t = -5^{\circ}\text{C}$; 2, -10°C ; 3, -20°C . P , MPa; τ , min.

$$PV_{\text{sol}} = Q_{\text{mel}}t/T_0. \quad (11)$$

The maximum of internal pressure $P_{\text{m.mel}} = Q_{\text{mel}}t/T_0(T_{\text{sol}} - V_{\text{liq}})$ is attained at $D = 1$ at the end of melting (destruction), whereas the minimum, equal to the external pressure $P = Q_{\text{mel}}t/T_0V_{\text{sol}}$, is attained $D = 0$ at the beginning of melting. The quantity $P_{\text{m.mel}}$ has the meaning of the instantaneous, greatest possible strength, i.e., the meaning of the pressure that a body can withstand without destruction (not being deformed above j_{mel}) at least for a moment $\tau_a \approx 10^{-13}$ sec. We will find the relationship between the internal and external pressures (we will divide one by the other) at $t = \text{const}$ and represent it in the form $P/(j_{\text{mel}}P_{\text{m.mel}}) = 1$. At $\tau_{1,t} = \tau_a$ we have

$$(\tau_{1,t}/\tau_a) = P/(j_{\text{mel}}P_{\text{m.mel}}). \quad (12)$$

By analogy with Eq. (12), at $\tau_{1,t} > \tau_a$ we will assume that the ratios of infinitely small increments of the longevity and the respective strength to their constant values are proportional. Then, having rewritten (12) in a differential form $d\tau/\tau = (dP/P)/j_{\text{mel}}$ and integrated it from τ_a to $\tau_{1,t}$ from the left and from P to $P_{\text{m.mel}}$ from the right, we obtain

$$\ln(\tau_a/\tau_{1,t}) = [\ln(P/P_{\text{m.mel}})]/j_{\text{mel}}. \quad (13)$$

After the transformation of (13), subject to (6), we find an equation of long-term strength (or longevity) of an ice body on plastic destruction with constant P and t that is identical to Eq. (4):

$$(P/P_{\text{m.mel}}) = (Pb/t) = (\tau_a/\tau_{1,t})^{j_{\text{mel}}}. \quad (14)$$

The exponent $j_{\text{mel}} = 0.083$ corresponds to the limiting state in which the specific volume of the liquid phase V in (6) is equal to V_{liq} and $b = 0.083^{\circ}\text{C}/\text{MPa}$. It is shown above that generally the limiting relative deformation is numerically approximately equal to b , without allowance for its dimensionality ($^{\circ}\text{C}/\text{MPa}$). Consequently, j_{mel} in (14) fluctuates, just as the numerical value of b , within the range 0.083–0.917. But in the majority of cases, both b and j_{mel} are closer to the lower limit of the interval [4].

Figure 3 presents the results of experimental determination of the strength of frozen grounds of different compositions (suspensions and clays) at different temperatures (-5°C , -10°C , and -20°C) and forms of stress (compression of

a suspension and cohesion of clay) according to S. S. Vyalov's data [2] (points on the graph) and of the calculation of the strength by Eq. (14) (lines). In the calculations, instead of the theoretical value $j_{\text{mel}} = 0.083$, the value of the limiting deformation j_{lim} found experimentally was substituted into Eq. (14). The latter quantity turned out to be a constant for the given type of a soil, equal to 0.115 for a suspension and 0.083 for clay (in this case, the values of j_{lim} and j_{mel} coincided). Despite the range of extrapolation, we may speak of a satisfactory coincidence of the results obtained from Eq. (14) with actual data.

The integral equation (14) reflects an isothermal-isochoric process. Having rewritten Eq. (12) in a differential form as $d\tau/\tau = (dV/V)P/P_{\text{m.mel}}$ and integrated it over τ from τ_a to $\tau_{1,t}$ on the left and over V from V_{sol} to V_{liq} on the right, we will find the integral equation of the isothermal-isobaric process:

$$\ln(\tau_{1,t}/\tau_a) = (P_{\text{m.mel}}/P) [\ln(V_{\text{liq}}/V_{\text{sol}})]. \quad (15)$$

If we take arbitrary values $\tau_a < \tau < \tau_{1,t}$ and $V_{\text{liq}} < V < V_{\text{sol}}$ as the upper limits of integration, we obtain

$$\ln(\tau/\tau_a) = (P_{\text{m.mel}}/P) [\ln(V/V_{\text{sol}})]. \quad (16)$$

Having divided (16) by (15) and taken into account that $\ln(V_{\text{liq}}/V_{\text{sol}}) \approx (V_{\text{liq}}/V_{\text{sol}}) - 1$, and the more so that $\ln(V/V_{\text{sol}}) \approx (V/V_{\text{sol}}) - 1$, we obtain, after transformation, an equation of the time dependence of deformation in plastic destruction identical to Eq. (5):

$$j/j_{\text{mel}} = \ln(\tau_{1,t}/\tau_a) / [\ln(\tau/\tau_a)]. \quad (17)$$

Calculations by the proposed equations demonstrate a generally satisfactory convergence with experimental results of [4, 9]; the limiting deformation in experiments is close to $j_{\text{mel}} = 0.083$, but it is always somewhat higher — fluctuates in the range 0.1–0.16, which coincides with the data of [2]. The discrepancy is due to the fact that the integrity of a frozen soil is provided not only by ice bonds but also by other ones. Moreover, the conceptual equation (1), just as (14), involves volumetric expenditures of phase heat, with these equations being obtained mainly from the analysis of tests for uniaxial rupture or compression. Therefore j_{mel} in (14) should be understood to mean (and j_{mel} should be replaced by) a certain effective longitudinal deformation j'_{mel} that allows for the nonuniform change in the body volume on one-sided compression (or extension) in phase transitions. Linking of the longitudinal deformation in experiments with the volumetric one is made by using the well-known relationships [2] between elastic constants: moduli of the longitudinal (E , the Young modulus) and volume (K_{el}) elasticities and the Poisson coefficient μ :

$$j'_{\text{mel}} = j_{\text{mel}} K_{\text{el}}/E = j_{\text{mel}}/[3(1-2\mu)]. \quad (18)$$

According to the data of various authors [8, 10], the Poisson coefficient of freshwater ice varies within 0.33–0.38. Then, the factor in the changing volume of melting ice j_{mel} in (18) that transforms it into an effective longitudinal deformation (K/E) will lie within the range 1.0–1.39, with the value of j'_{mel} itself being equal to 0.083–0.12.

It can be easily proved that the validity of Eqs. (14) and (17) will not be disturbed if instead of the instantaneous values of the longevity and strength we take, as the base (initial) ones, the first measured pair: τ_{con} and P_{con} , imparting to them the status of conventionally instantaneous longevity and strength. Moreover, in engineering calculations it is advisable to replace the theoretical value of the limiting deformation equal to j_{mel} by its actual value found in the process of standard creep tests, i.e., j_{lim} . Then (14) acquires a simple form convenient for calculations:

$$P = P_{\text{con}} (\tau/\tau_{\text{con}})^{j_{\text{lim}}}. \quad (19)$$

Table 1 presents examples of a satisfactory coincidence between actual data and calculations by Eqs. (14) and (19). As actual data, the results of experiments carried out by A. P. Kardymon (tests 1 and 2) and by Yu. Ya. Velli and V. N. Ivanov (tests 3 and 4) were used after they had been reduced and processed in review [4]. In the former case, A. P. Kardymon determined the resistance of frozen sand to shear along the side surface of metal models of piles of rectangular (tests 1) and square (tests 2) cross section at $t = -1.3^\circ\text{C}$; in the latter case, the resistance of frozen

TABLE 1. Actual Values of P_{act} and Values of the Resistance of Frozen Soil to Shear P_{14} and P_{19} Calculated from Eqs. (14) and (19), Respectively, for Different Longevities $t_{l,t}$

$t_{l,t}$, min	P_{act} , MPa	P_{14} , MPa	P_{19} , MPa	P_{act} , MPa	P_{14} , MPa	P_{19} , MPa
<i>Test 1</i>			<i>Test 2</i>			
30	0.37	0.37	0.37	0.45	0.45	0.45
60	0.35	0.35	0.34	0.43	0.43	0.42
120	0.33	0.32	0.32	0.40	0.40	0.40
180	—	0.31	0.31	0.39	0.39	0.39
520	—	0.29	0.29	0.35	0.35	0.35
<i>Test 3</i>			<i>Test 4</i>			
42	0.44	0.45	0.45	0.51	0.55	0.55
60	0.40	0.43	0.42	0.49	0.52	0.49
222	0.37	0.36	0.36	0.45	0.50	0.42
420	0.34	0.33	0.33	0.44	0.42	0.38
600	0.31	0.32	0.31	0.43	0.41	0.37

loam to shear along the side surface of round metal (tests 3) and concrete (tests 4) models of piles at $t = -4.5^{\circ}\text{C}$. The values of j_{lim} calculated from the results of four series of tests were respectively equal to 0.1, 0.095, 0.128, and 0.124, values close to those of j_{mel} .

The thawing of frozen soils (melting of ice bonds) at ordinary temperatures corresponds to their compression; the melting of the majority of other ("normal") solid bodies corresponds to extension. In the models considered, the vectors of mechanical deformation and of the deformation due to phase conversions coincide, aggravating the process of destruction and imparting a resonance character to it. On compression of frozen soils and ice, which is close to the conditions of their operation as bases of engineering constructions, their thawing and secondary phase compression occur; on extension of "normal" solid bodies, sublimation (melting + evaporation) and secondary phase extension occur.

At first glance, the reason for the destruction of frozen soil on extension cannot be the melting of ice (decrease in the ice content in the ground), since on extension the pore pressure decreases and the ice content, vice versa, must increase due to the freezing of bound water. But the longitudinal (uniaxial) extension of an ice body is accompanied by its compression and, correspondingly, by melting in the transverse direction in the region of maximum contraction and stress concentration — in the "neck," precisely along the central section of which rupture occurs (Fig. 1e). This section is the axial plane of transverse symmetry, where the field of stresses and deformations is the same as in a radially compressed infinite cylinder, i.e., it is reduced to a one-dimensional field, when the radial deformation is equal to the square root of the bulk deformation (in the specific section, of the areal one) and, consequently, the 8–10% contraction of the volume needed for the "liquefaction" of ice bonds corresponds to the 3–3.5% decrease in the radius of the cylinder. An analysis of experimental data [2, 3] has shown that in the case of longitudinal extension of samples of frozen soil, the coupling between the long-term strength and longevity is also described by empirical equation (19), with the value of j_{lim} varying approximately in the same range as in the case of longitudinal compression [4]. This means that in the case of uniaxial extension, as well as compression, the destruction of a frozen soil begins at values of longitudinal deformation close to 0.1 (on extension the value of j_{lim} is somewhat higher than on compression).

Conclusions. One can distinguish two basic forms of destruction of a solid body: a slow one manifesting itself as a gradual increase in external deformation and a jumpwise, brittle one developing due to internal formation of cracks. The former is completed by transition into a liquid-like state, and the latter, by opening of cracks and disintegration into parts. Both forms of destruction are related to phase transitions. The former, typical of frozen soils and ice, operates near the melting point, and the latter, sublimation, far from it. The proposed equations (2)–(6) and (19) comprise a full description of the unified process of deformation and destruction of frozen soil under constant loading; this description is suitable for determining the strength and longevity as well as deformation as a function of time.

NOTATION

b , coefficient of commensurability between pressure and melting temperature, $^{\circ}\text{C}/\text{MPa}$; D , degree of pressing on unfrozen water, fractions of unity; E , modulus of longitudinal elasticity (Young's modulus), MPa; j_{mel} , change in body volume (volumetric relative deformation) in melting, fractions of unity; j_{crys} , same in crystallization; j_{cr} , volumet-

ric relative deformation at the moment of opening of a crack; j_{lim} , limiting (destructive) deformation determined experimentally; j'_{mel} , effective longitudinal deformation; K_{el} , modulus of volumetric elasticity, MPa; L_{crys} , volumetric heat of crystallization, kJ/m³; L_{mel} , same of melting; P , external pressure, MPa; P_{por} , same porous one; $P_{m.sub}$, maximum strength in brittle (sublimation) destruction, MPa; $P_{m.mel}$, same in plastic destruction; Q_{sub} , latent heat of sublimation, kJ/mole; Q_{mel} , same of melting; Q_{ev} , same of evaporation; R , universal gas constant, kJ/(mole·T); T , temperature, K; t , same, °C; T_{mel} , melting temperature, K; T_0 , same at atmospheric pressure; T_{eut} , eutectic temperature, K; U , energy of destruction activation, kJ/mole; V , current specific volume, m³/kg; T_{sol} , specific volume of a solid phase, m³/kg; V_{liq} , same of a liquid phase; γ , thermofluctuational volume of an atom, kJ/(mole·MPa); η , coefficient of averaging, fractions of unity; μ , Poisson coefficient, fractions of unity; τ , time, sec; $\tau_{l.t.}$, longevity, sec; τ_{max} , maximum longevity, sec; τ_a , period of thermal vibrations of atoms, sec; τ_{con} , conventionally instantaneous longevity, sec. Subscripts and superscripts: act, actual; a, atom; con, conventional; cr, crack; crys, crystallization; el, elasticity; eut, eutectics; ev, evaporation; lim, limiting; liq, liquid; l.t, long-term, sub, sublimation; max, maximum; mel, melting; por, porous; sol, solid phase; .

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